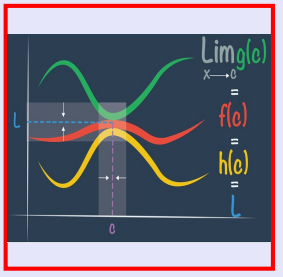
Calculus I

Lecture 28



Feb 19-8:47 AM

Find $\frac{dy}{dx}$

$y\cos x = x^2 + y^2$

$\frac{d}{dx}[y\cos x] = \frac{d}{dx}[x^2 + y^2]$

product

$\frac{dy}{dx}\cdot\cos x + y\cdot\frac{d}{dx}[\cos x] = \frac{d}{dx}[x^2] + \frac{d}{dx}[y^2]$

$\frac{dy}{dx}\cdot\cos x + y\cdot -\sin x = 2x + 2y\frac{dy}{dx}$

$[\cos x - 2y]\frac{dy}{dx} = 2x + y\sin x$

$$\frac{dy}{dx} = \frac{2x + y\sin x}{\cos x - 2y}$$

Oct 16-7:26 AM

Find $\frac{dy}{dx}$

$$y \sin x^2 = x \sin y^2$$

$$\frac{d}{dx}[y \sin x^2] = \frac{d}{dx}[x \sin y^2]$$

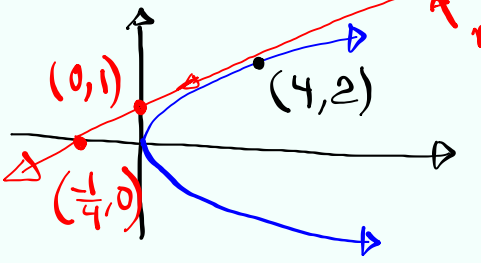
$$\frac{dy}{dx} \cdot \sin x^2 + y \cdot \cos x^2 \cdot 2x = 1 \cdot \sin y^2 + x \cdot \cos y^2 \cdot 2y \cdot \frac{dy}{dx}$$

$$(\sin x^2 - 2xy\cos y^2)\frac{dy}{dx} = \sin y^2 - 2xy\cos x^2$$

$$\frac{dy}{dx} = \frac{\sin y^2 - 2xy\cos x^2}{\sin x^2 - 2xy\cos y^2}$$

Oct 16-7:32 AM

Find eqn of the tan. line to the graph of $y^2 = x$ at $(4,2)$.



$$m = \frac{dy}{dx}\Big|_{(4,2)} = \frac{1}{2(2)} = \frac{1}{4}$$

$$\frac{d}{dx}[y^2] = \frac{d}{dx}[x]$$

$$2y\frac{dy}{dx} = 1$$

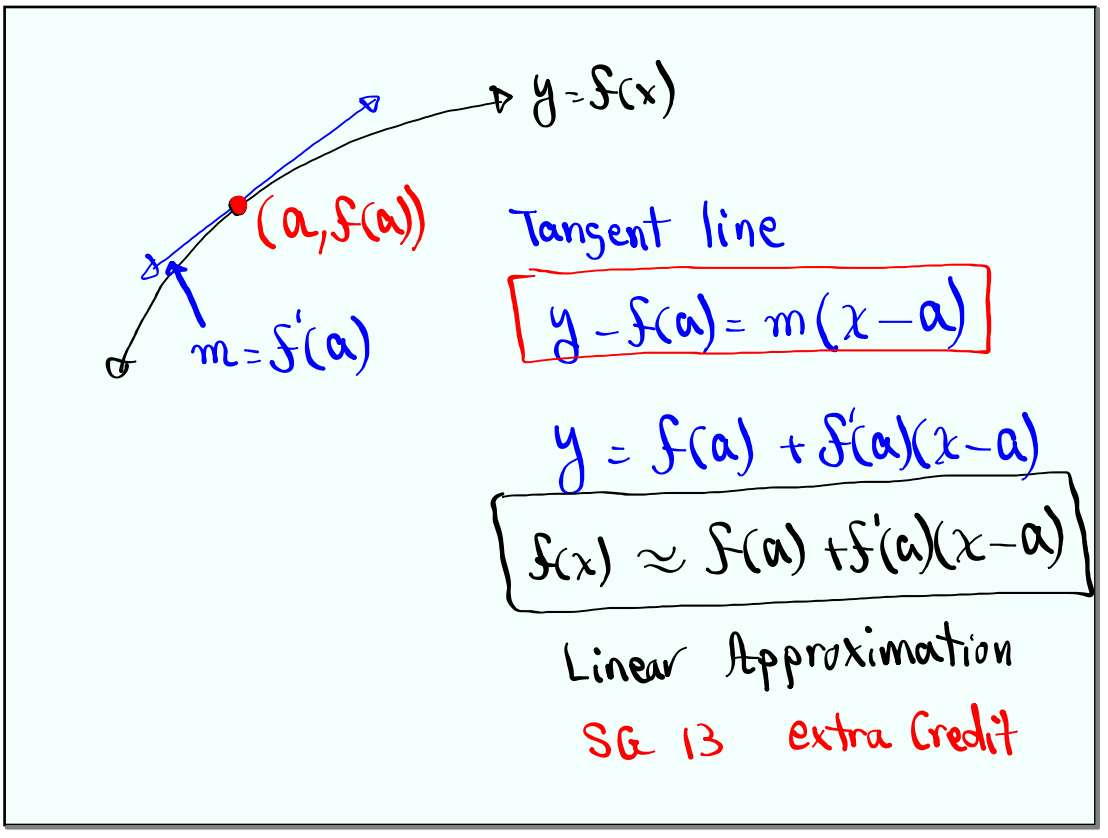
$$\frac{dy}{dx} = \frac{1}{2y}$$

$$y - 2 = \frac{1}{4}(x-4)$$

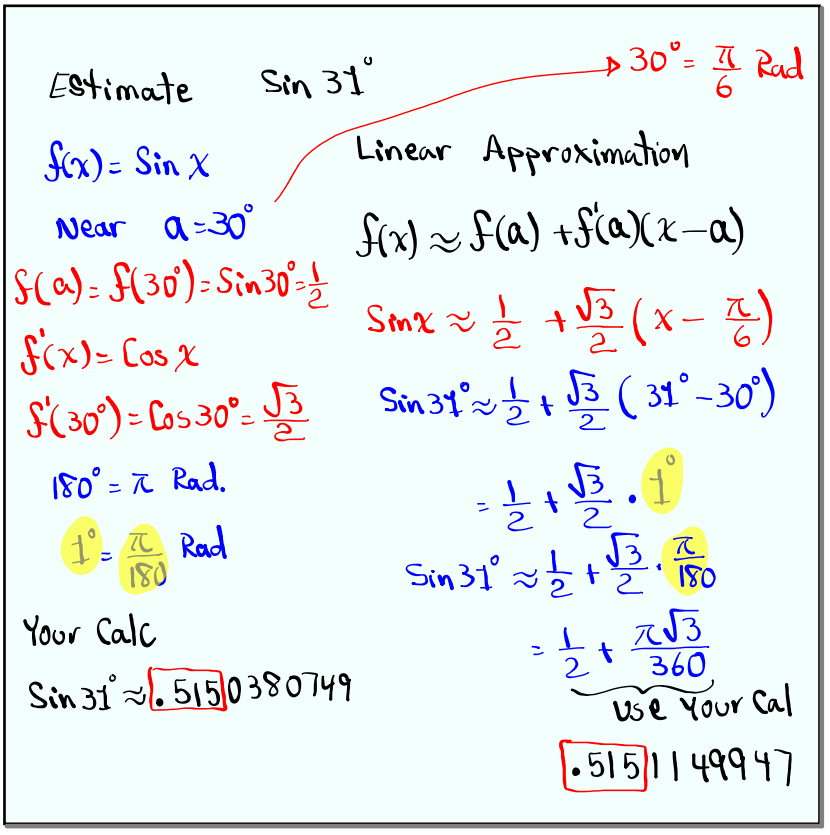
$$y - 2 = \frac{1}{4}x - 1$$

$$y = \frac{1}{4}x + 1$$

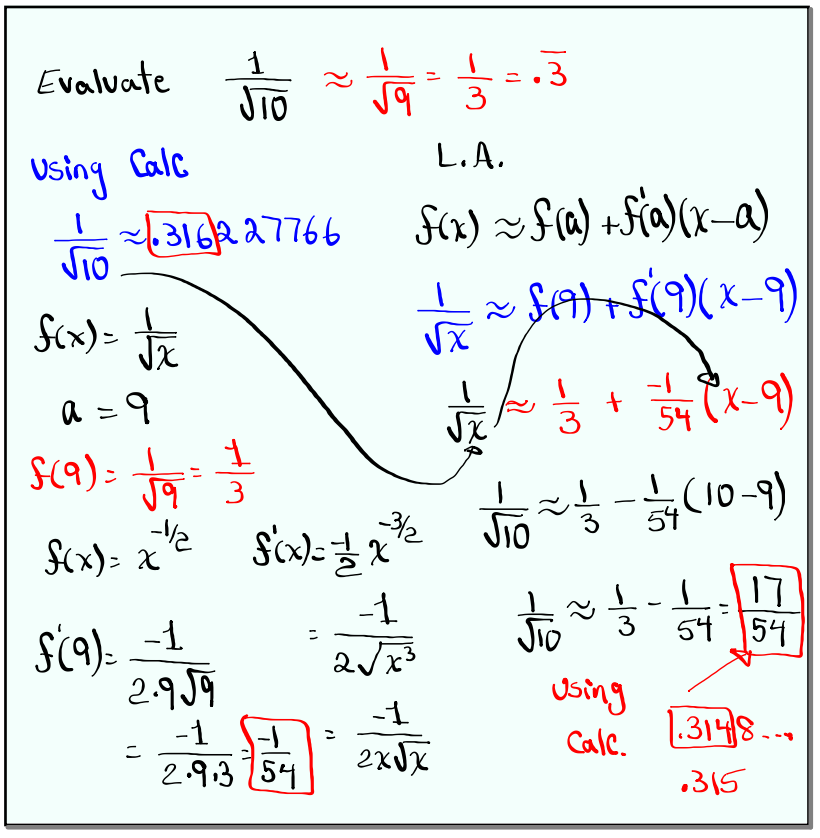
Oct 16-7:40 AM



Oct 15-8:27 AM



Oct 16-7:46 AM



Oct 16-7:54 AM

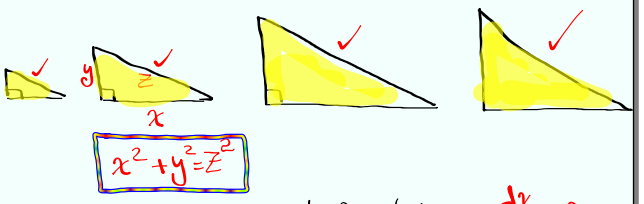
Quadratic Approximation

$$f(x) \approx \underbrace{f(a) + f'(a)(x-a)}_{\text{Linear}} + \frac{f''(a)}{2}(x-a)^2$$

$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{Quadratic}}$

Oct 16-8:02 AM

Related Rates:



$x^2+y^2=z^2$

x increases at 2 cm/min $\Rightarrow \frac{dx}{dt}=2$

y increases at 3 cm/min $\Rightarrow \frac{dy}{dt}=3$

How fast the hyp. increase when $x=3$ & $y=4$?

$x^2+y^2=z^2$

$\frac{d}{dt}\left[x^2+y^2\right]=\frac{d}{dt}\left[z^2\right]$

$2x\cdot\frac{dx}{dt}+2y\cdot\frac{dy}{dt}=2z\cdot\frac{dz}{dt}$

$x^2+y^2=z^2$

$3^2+4^2=z^2$

$z=5$

$3\cdot 2+4\cdot 3=5\cdot\frac{dz}{dt}$

$6+12=5\frac{dz}{dt}$

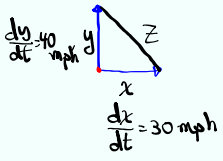
$\frac{dz}{dt}=\frac{18}{5}$ cm/min.

Oct 16-8:05 AM

Dayane & Freddy leave same intersection at the same time.

Dayane goes east at 30 mph,
Freddy goes north at 40 mph.

How fast is the distance between them change in one hour?

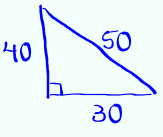


$x^2+y^2=z^2$

$\frac{d}{dt}\left[x^2+y^2\right]=\frac{d}{dt}\left[z^2\right]$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$

1 hr



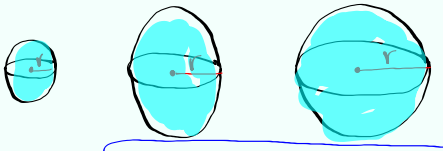
$30\cdot 30+40\cdot 40=50\cdot\frac{dz}{dt}$

$900+1600=50\frac{dz}{dt}$

$\frac{2500}{50}=\frac{dz}{dt}$ $\quad\frac{dz}{dt}=50$ mph

Oct 16-8:14 AM

You are pumping air into a ball.



Suppose its radius increases at 2cm/min. $\frac{dr}{dt} = 2$

How fast the volume increases when the radius is 5cm?

Volume of Sphere $V = \frac{4\pi r^3}{3}$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4\pi r^3}{3}\right]$$

$$\frac{dV}{dt} = \frac{4\pi}{3} \cdot 3r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi \cdot 5^2 \cdot 2 = 200\pi \text{ cm}^3/\text{min}$$

Oct 16-8:22 AM