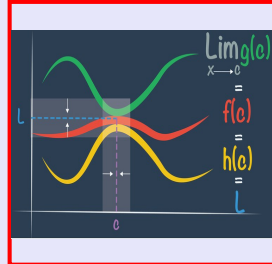


# Calculus I

## Lecture 28



Feb 19-8:47 AM

find  $\frac{dy}{dx}$

$$y \cos x = x^2 + y^2$$

$$\frac{d}{dx} [y \cos x] = \frac{d}{dx} [x^2 + y^2]$$

product

$$\frac{dy}{dx} \cdot \cos x + y \cdot \frac{d}{dx} [\cos x] = \frac{d}{dx} [x^2] + \frac{d}{dx} [y^2]$$

$$\frac{dy}{dx} \cdot \cos x + y \cdot (-\sin x) = 2x + 2y \frac{dy}{dx}$$

$$[\cos x - 2y] \frac{dy}{dx} = 2x + y \sin x$$

$$\frac{dy}{dx} = \frac{2x + y \sin x}{\cos x - 2y}$$

Oct 16-7:26 AM

Find  $\frac{dy}{dx}$

$$y \sin x^2 = x \sin y^2$$

$$\frac{d}{dx} [y \sin x^2] = \frac{d}{dx} [x \sin y^2]$$

$$\frac{dy}{dx} \cdot \sin x^2 + y \cdot \cos x^2 \cdot 2x = 1 \cdot \sin y^2 + x \cdot \cos y^2 \cdot 2y \cdot \frac{dy}{dx}$$

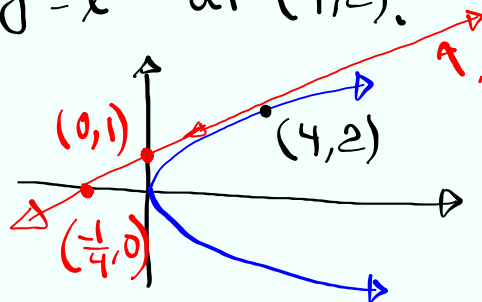
$$(\sin x^2 - 2xy \cos y^2) \frac{dy}{dx} = \sin y^2 - 2xy \cos x^2$$

$$\frac{dy}{dx} = \frac{\sin y^2 - 2xy \cos x^2}{\sin x^2 - 2xy \cos y^2}$$

Oct 16-7:32 AM

Find eqn of the tan. line to the graph of

$$y^2 = x \text{ at } (4, 2).$$



$$m = \frac{dy}{dx} \Big|_{(4,2)} = \frac{1}{2(2)} = \frac{1}{4}$$

$$\frac{d}{dx} [y^2] = \frac{d}{dx} [x]$$

$$2y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

$$y - 2 = \frac{1}{4}(x - 4)$$

$$y - 2 = \frac{1}{4}x - 1$$

$$y = \frac{1}{4}x + 1$$

Oct 16-7:40 AM

$y = f(x)$

$(a, f(a))$

$m = f'(a)$

Tangent line

$y - f(a) = m(x - a)$

$y = f(a) + f'(a)(x - a)$

$f(x) \approx f(a) + f'(a)(x - a)$

Linear Approximation

SG 13 extra Credit

Oct 15-8:27 AM

Estimate  $\sin 31^\circ$   $\rightarrow 30^\circ = \frac{\pi}{6}$  Rad

$f(x) = \sin x$  Linear Approximation

Near  $a = 30^\circ$

$f(a) = f(30^\circ) = \sin 30^\circ = \frac{1}{2}$

$f'(x) = \cos x$

$f'(30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$180^\circ = \pi$  Rad

$1^\circ = \frac{\pi}{180}$  Rad

Your Calc

$\sin 31^\circ \approx \boxed{.5150380749}$

$f(x) \approx f(a) + f'(a)(x - a)$

$\sin x \approx \frac{1}{2} + \frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$

$\sin 31^\circ \approx \frac{1}{2} + \frac{\sqrt{3}}{2}(31^\circ - 30^\circ)$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot 1^\circ$

$\sin 31^\circ \approx \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\pi}{180}$

$= \frac{1}{2} + \frac{\pi\sqrt{3}}{360}$

use your Cal

$\boxed{.5151149947}$

Oct 16-7:46 AM

Evaluate  $\frac{1}{\sqrt{10}} \approx \frac{1}{\sqrt{9}} = \frac{1}{3} = .\bar{3}$

Using Calc L.A.

$\frac{1}{\sqrt{10}} \approx \boxed{.316227766}$   $f(x) \approx f(a) + f'(a)(x-a)$

$f(x) = \frac{1}{\sqrt{x}}$   $\frac{1}{\sqrt{x}} \approx f(9) + f'(9)(x-9)$

$a = 9$   $\frac{1}{\sqrt{x}} \approx \frac{1}{3} + \frac{-1}{54}(x-9)$

$f(9) = \frac{1}{\sqrt{9}} = \frac{1}{3}$   $\frac{1}{\sqrt{10}} \approx \frac{1}{3} - \frac{1}{54}(10-9)$

$f(x) = x^{-1/2}$   $f'(x) = -\frac{1}{2}x^{-3/2}$

$f'(9) = \frac{-1}{2 \cdot 9 \sqrt{9}} = \frac{-1}{2 \cdot 9 \cdot 3} = \frac{-1}{54}$   $\frac{1}{\sqrt{10}} \approx \frac{1}{3} - \frac{1}{54} = \frac{17}{54}$

Using Calc.  $\boxed{.3148...}$   
. $315$

Oct 16-7:54 AM

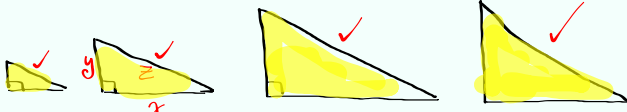
Quadratic Approximation

$$f(x) \approx \underbrace{f(a) + f'(a)(x-a)}_{\text{Linear}} + \frac{f''(a)}{2}(x-a)^2$$

Quadratic

Oct 16-8:02 AM

Related Rates:



$$x^2 + y^2 = z^2$$

$x$  increases at  $2 \text{ cm/min} \Rightarrow \frac{dx}{dt} = 2$   
 $y$  increases at  $3 \text{ cm/min} \Rightarrow \frac{dy}{dt} = 3$

How fast the hyp. increase when  $x=3$  &  $y=4$ ?

$$x^2 + y^2 = z^2$$

$$\frac{d}{dt}[x^2 + y^2] = \frac{d}{dt}[z^2]$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$$

$z=5$

$$3 \cdot 2 + 4 \cdot 3 = 5 \cdot \frac{dz}{dt}$$

$$6 + 12 = 5 \frac{dz}{dt}$$

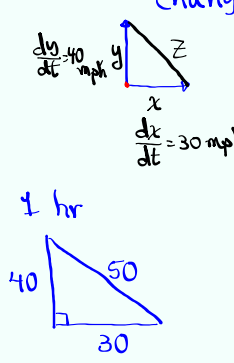
$$\frac{dz}{dt} = \frac{18}{5} \text{ cm/min.}$$

Oct 16-8:05 AM

Dayane & Freddy leave same intersection at the same time.

Dayane goes east at  $30 \text{ mph}$ ,  
 Freddy goes north at  $40 \text{ mph}$ .

How fast is the distance between them change in one hour?



$$x^2 + y^2 = z^2$$

$$\frac{d}{dt}[x^2 + y^2] = \frac{d}{dt}[z^2]$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$30 \cdot 30 + 40 \cdot 40 = 50 \cdot \frac{dz}{dt}$$

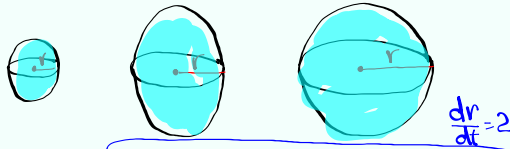
$$900 + 1600 = 50 \frac{dz}{dt}$$

$$\frac{2500}{50} = \frac{dz}{dt} \quad \frac{dz}{dt} = 50 \text{ mph}$$

1 hr

Oct 16-8:14 AM

You are pumping air into a ball.



Suppose its radius increases at  $2 \text{ cm/min}$ .

How fast the volume increases when the radius is  $5 \text{ cm}$ ?

Volume of Sphere  $V = \frac{4\pi r^3}{3}$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4\pi r^3}{3}\right]$$

$$\frac{dV}{dt} = \frac{4\pi}{3} \cdot 3r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi \cdot 5^2 \cdot 2 = 200\pi \text{ cm}^3/\text{min}$$

Oct 16-8:22 AM